Boolean Algebra

Dr. Monika Patel
Assistant Professor
Dept of Computer Science
Durga Mahavidyalaya
Raipur (CG)

1. Introduction

Boolean Algebra is a branch of mathematics that deals with binary variables and logic operations. Named after George Boole, it forms the foundation of digital electronics and computer science. Boolean algebra simplifies the analysis and design of digital circuits by using variables that assume only two values: 0 and 1.

2. Boolean Variables and Values

In Boolean algebra, variables represent logical statements and can take only two possible values:

- 1 (TRUE)
- 0 (FALSE)

These values are used in logical operations to model decision-making in circuits.

3. Basic Boolean Operations

There are three primary operations in Boolean algebra:

- AND (· or ∧): Outputs 1 if both inputs are 1.

Truth Table:

A B | A·B

0 0 | 0

0 1 | 0

1 0 | 0

1 1 | 1

- OR (+ or ∨): Outputs 1 if at least one input is 1.

Truth Table:

A B | A+B

0 0 | 0

0 1 | 1

1 0 | 1

1 1 | 1

- NOT (' or ¬): Outputs the opposite of the input.

Truth Table:

A | A'

0 | 1

1 | 0

4. Boolean Laws and Properties

Boolean algebra follows specific rules:

- Identity Laws:

A + 0 = A, A · 1 = A

- Null Laws:

A + 1 = 1, A · 0 = 0

- Idempotent Laws:

$A + A = A$, $A \cdot A = A$

- Complement Laws:

$A + A' = 1$, $A \cdot A' = 0$

- Involution Law:

$(A')' = A$

- Commutative Laws:

$A + B = B + A$, $A \cdot B = B \cdot A$

- Associative Laws:

$A + (B + C) = (A + B) + C$

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$

- Distributive Laws:

$A \cdot (B + C) = A \cdot B + A \cdot C$

$A + (B \cdot C) = (A + B) \cdot (A + C)$

- De Morgan’s Theorems:

$(A \cdot B)' = A' + B'$

$(A + B)' = A' \cdot B'$

5. Duality Principle

The duality principle states that every algebraic expression remains valid if the operators and identity elements are interchanged:

- Replace AND (·) with OR (+)

- Replace OR (+) with AND (·)

- Replace 0 with 1, and 1 with 0

Example:

Original: $A + 0 = A$

Dual: $A \cdot 1 = A$

6. Standard Forms

Standard forms are used to express Boolean functions:

- Sum of Products (SOP):

An OR of multiple AND terms (e.g., $A'B + AB' + ABC$).

- Product of Sums (POS):

An AND of multiple OR terms (e.g., $(A + B)(A' + C)$).

7. Applications

Boolean algebra is widely used in:

- Digital circuit design

- Simplifying logic expressions

- Programming and database queries

- Artificial intelligence and decision systems

8. Karnaugh Map (K-Map)

K-Maps are a visual method to simplify Boolean expressions by grouping adjacent 1s.

They help minimize logic functions without algebraic methods.

9. Logic Gates Summary

Logic gates implement Boolean functions:

- AND Gate: Output is 1 if all inputs are 1
- OR Gate: Output is 1 if at least one input is 1
- NOT Gate: Output is the inverse of the input
- NAND Gate: Inverse of AND
- NOR Gate: Inverse of OR
- XOR Gate: Output is 1 if inputs are different
- XNOR Gate: Output is 1 if inputs are the same